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Technical Notes

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Critical Coating Radius of Spherical Particle in Thermal Radiation Environment

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Nomenclature

Bi	=	Biot number, hr_i/k
h	=	heat transfer coefficient outside the coating, $\mathrm{W/m^2\,K}$
k	=	coating thermal conductivity, W/m K
Q	=	dimensionless total heat transfer
Q_c	=	dimensionless convection heat transfer
Q_k	=	dimensionless conduction heat transfer
$Q_{\max}$	=	dimensionless maximum heat transfer
Q_r	=	dimensionless radiation heat transfer
q	=	total heat transfer, W
R	=	dimensionless coating outer radius, r_o/r_i
R_{cr}	=	dimensionless critical radius coating outer radius, $r_{o,\mathrm{cr}}/r_i$
r_i	=	inner coating radius, m
r_o	=	outer coating radius, m
T_i	=	temperature at the inner surface of coating, K
T_o	=	temperature at the outer surface of coating, K
T_∞	=	temperature of the surroundings, K

Greek symbols

β	=	dimensionless radiation parameter, $\varepsilon\sigma T_i^3 r_i/k$
ε	=	emissivity
θ_o	=	dimensionless temperature at the outer surface of coating, T_o/T_i
θ_∞	=	dimensionless temperature of the surroundings, T_∞/T_i
σ	=	Stefan-Boltzman constant

Subscripts

cr	=	critical
i	=	inner coating surface
o	=	outer coating surface
max	=	maximum

I. Introduction

THE critical thickness of insulation or coating is very important in many thermal applications. This paper presents the critical thickness for a sphere in a radiation and convection environment. The authors found that the critical thickness of insulation for a sphere with radiation effects has not been investigated in the literature. From the literature, it is clear that the critical thickness of insulation for a cylinder with radiation [1] and a sphere without radiation [2] are available, and so it is necessary to investigate the case of a sphere with radiation and report the findings.

The applications for a spherical surface which is coated with layers of different material or insulation, and in a radiation and convection environment, has wide application in industry. Industrial application includes use in combustion of powdered coal or pulverized coal covered with an ash layer, heat transfer in micro ball bearings coated with a high hardness layer, and insulation of a melting pot. In nanopaticle thermotherapy [3], convection and conduction heat transfer to nearby tissue is the dominant mechanism of heat transfer, where maximum heat transfer is a function of external convection, coating thermal conductivity, and coating thickness.

Scientists are interested in coating nanoparticles with different material films. Such coatings have a strong effect on nanoparticle applications. It is very important to control and understand the temperature of nanoparticles when used in thermotherapy [3]. Nanoparticle thermotherapy is known as magnetic nanoparticle therapy where induced localized hyperthermia is used to kill or weaken tumor cells. The ability to manipulate the surface properties of the nanoparticles through deposition of one or more material layers can greatly enhance their effectiveness. The variation in coating thickness could have a big influence on temperature distribution within a nanoparticle fluid. Different coating materials are used in nanoparticle thermotherapy to assure a homogenous low viscosity fluid allowing good penetration of the tumor cell with late detection by the immune system [3]. Magnetite nanoparticles, surface-coated with dextran or dimercaptosuccinic acid (DMSA), have been successfully used to produce biocompatible magnetic fluids (BMFs) [3]. Johannsen et al. [4] found that thermotherapy using magnetic nanoparticles is a feasible procedure in treating prostate cancer.

The use of the critical radius for radial heat conduction in thermal insulation systems has been widely reported in the literature [1,5–7]. One of the early works that investigated the critical thickness using a numerical solution was presented by Simmons [8]. An optimum distribution of a finite amount of insulating material over a nonisothermal wall was studied by Bejan [9], to minimize the total heat loss from the wall to the surroundings.

Recently, scientists considered the effects of a biocompatible coating layer on the magnetic properties of superparamagnetic iron oxide nanoparticles [10,11]. Superparamagnetic particles are nanosized particles that change the direction of magnetization of the entire crystallite due to thermal energy. Similarly, it is important to understand the effect of such coatings on the heat transfer and temperature

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distribution within a nanoparticle fluid mainly used for thermotherapy applications. Thermotherapy is used as a cancer treatment to kill or weaken tumor cells, with negligible effects on healthy cells [10,11]. Tumor cells, with a disorganized and compact vascular structure, have difficulty dissipating heat. Therefore, hyperthermia may cause cancerous cells to undergo apoptosis (i.e., cell self-destruction) due to the applied heat, whereas healthy cells can more easily maintain a normal temperature by better heat dissipation. Metal nanoparticles efficiently generate heat in the presence of electromagnetic radiation. In this study, the authors present the effect of a coating on heat transfer from a single spherical particle.

Sahin and Kalyon [1] presented an analytical study on the critical radius of insulation for a circular tube subjected to radiative and convective heat transfer. They presented three closed form solutions for insulation in cylindrical coordinates. Lee et al. [12] used regular polygon top solid wedge thermal resistance to characterize the heat transfer for an insulated regular polyhedron. Lee et al. showed that the thermal resistance of the inner convection term and the body conduction term cannot be neglected, especially in situations with small body sizes or large outer convection coefficients.

Kulkarni [13] introduced the crossover radius to show that critical radius for radial heat conduction is a necessary criterion for increasing heat dissipation but not always sufficient as in insulation. Al-Nimr and Abdallah [14] presented an analytical model using the Green's function method to study the critical thickness for transient insulated electric wire producing pulsating signals.

II. Problem Formulation

Figure 1 represents the investigated problem schematic with its coordinate system. A three-dimensional sphere is shown with a coating (or insulation) layer. The coating layer has thermal conductivity of k. The inner and outer surface radii are defined as r_i and r_o, respectively. The sphere is assumed to be a lumped system with an inner temperature of T_i. The outer surface temperature of the coating is defined as T_o. Heat is conducted though the coating to the coating outer surface where it is convected and radiated to the surroundings at T_∞. The aim is to find the optimum thickness, $r_o - r_i$, and achieve maximum heat transfer for a spherical coated particle.

The following dimensionless variables were used:

$$Q = \frac{q}{4\pi r_i k T_i}, \qquad R = \frac{r_o}{r_i}, \qquad \theta_o = \frac{T_o}{T_i}, \qquad \theta_\infty = \frac{T_\infty}{T_i} \tag{1}$$

The dimensionless steady state heat transfer across a spherical coating layer is modeled using a one-dimensional conduction model,

$$Q_k = \frac{(1-\theta_o)}{1-\frac{1}{R}} \tag{2}$$

The heat transfer from the outer surface by convection is modeled by a convection model,

$$Q_c = BiR^2(\theta_o - \theta_\infty) \tag{3}$$

and a radiation model

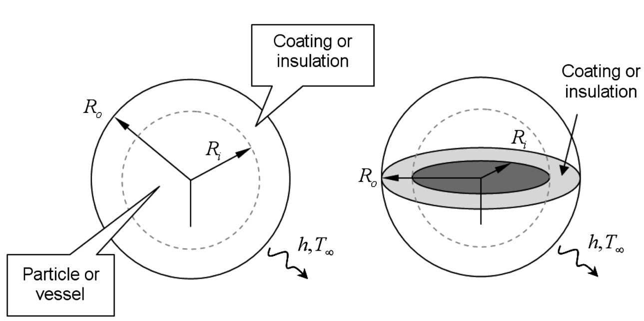

Fig. 1 Schematic diagram of a three-dimensional sphere with a shell thickness representing the coating thickness (or insulation thickness).

$$Q_r = \beta R^2(\theta_o^4 - \theta_\infty^4) \tag{4}$$

where

$$Q = Q_k = Q_c + Q_r \tag{5}$$

To find the relationship between Q and R, Eqs. (2–5) can be solved numerically for the four unknowns Q_k, Q_c, Q_r, and θ_o. In the current study, the authors present an analytical solution for three specific cases and numerical results for the general case.

III. Analytical Solution

Equations (2–5) can be reduced to two equations as follows:

$$Q = BiR^2(\theta_o - \theta_\infty) + \beta R^2(\theta_o^4 - \theta_\infty^4) \tag{6}$$

$$\frac{(1-\theta_o)}{1-(1/R)} = BiR^2(\theta_o - \theta_\infty) + \beta R^2(\theta_o^4 - \theta_\infty^4) \tag{7}$$

By differentiating Eq. (6) and (7) with respect to R and by solving for $\frac{dQ}{dR}$ the following equation is derived:

$$\frac{dQ}{dR} = \frac{[R^2 Bi(\theta_o - \theta_\infty) + R^2\beta Bi(\theta_o^4 - \theta_\infty^4)][(2/R) - (Bi + 4\beta\theta_o^3)]}{1 + (R^2 - R)(Bi + 4\beta\theta_o^3)} \tag{8}$$

The maximum heat transfer with respect to coating radius can be found by setting Eq. (8) equal to zero for the critical radius $R = R_{\mathrm{cr}}$. Equation (8) is equal to zero when $\theta_o = \theta_\infty$, which is a trivial solution, or when

$$R_{\mathrm{cr}} = \frac{2}{Bi + 4\beta\theta_o^3} \tag{9}$$

The dimensionless temperature θ_o of the coating outer surface is the root of Eq. (10), which is obtained by substituting Eq. (9) into Eq. (7):

$$1 - \theta_o = \left(\left(\frac{2}{Bi + 4\beta\theta_o^3}\right)^2 - \left(\frac{2}{Bi + 4\beta\theta_o^3}\right)\right) \times [Bi(\theta_o - \theta_\infty) + \beta(\theta_o^4 - \theta_\infty^4)] \tag{10}$$

By substituting different physically meaningful coating radius values $R = \frac{r_o}{r_i} > 1$ in Eq. (6), one can find that, at $R = R_{\mathrm{cr}}$, the maximum heat transfer $Q_{\max}$ is achieved, not the minimum heat transfer.

IV. Results and Discussion

As shown in the previous section, the maximum heat transfer can be achieved at the critical coating thickness. Determining the optimum coating thickness is essential to achieve the desired nanoparticle performance where more heat is used to raise the temperature of the cancer cell rather than increasing the temperature of the nanoparticle. This coating thickness is found to be a function of β, Bi, and θ_∞ as can be seen from Eqs. (7) and (9). Equations (7) and (9) can be used to calculate the critical thickness of an ash layer on pulverized coal particles which is critical for complete combustion of pulverized coal. Some special cases for critical thickness and maximum heat transfer are discussed in the next sections.

A. Case 1: Convection Only ($\beta = 0$)

By neglecting radiation, the critical coating thickness according to Eq. (9) can be simplified as follows:

$$R_{\mathrm{cr}} = \frac{2}{Bi} \tag{11}$$

The maximum heat transfer and the surface temperature are determined by substituting Eq. (11) into Eqs. (6) and (7):

$$Q_{\max} = Q|_{R_{\mathrm{cr}}} = \frac{(4/Bi)(1-\theta_\infty)}{1+((4/Bi)-2)} = \left(\frac{2R_{\mathrm{cr}}}{2R_{\mathrm{cr}}-1}\right)(1-\theta_\infty) \quad (12)$$

$$\theta_o|_{R_{\mathrm{cr}}} = \frac{1+\theta_\infty((4/Bi)-2)}{1+((4/Bi)-2)} = \frac{1+2\theta_\infty(R_{\mathrm{cr}}-1)}{1+2(R_{\mathrm{cr}}-1)} \quad (13)$$

The maximum heat transfer exists for $R_{\mathrm{cr}} > 1$ which means Bi should be less than two. For the limiting case where $R = 1$ or $Bi = 2$ (i.e., zero coating thickness), the surface temperature and maximum heat transfer become $\theta_o|_{R=1} = 1$ and $Q|_{R=1} = 2(1-\theta_\infty)$.

Figure 2 shows the variation of the heat transfer as a function of the coating thickness for different values of Bi and $\theta_\infty = 0.8$. The locus of the maximum heat transfer values is added to the figure as a dashed line.

B. Case 2: Radiation Only ($Bi = 0$)

In this case, radiation is considered to be the dominant mode of heat transfer, and therefore convection is neglected (i.e., $Bi = 0$). The radiation case does not occur in hyperthermia because convection and conduction are the dominant heat transfer modes, however the radiation only case is significant for space applications. The critical coating thickness in this case, according to Eq. (9) is

$$R_{\mathrm{cr}} = \frac{1}{2\beta\theta_o^3} \quad (14)$$

The maximum heat transfer and the surface temperature are determined by substituting Eq. (14) into Eqs. (6) and (7):

$$1-\theta_o|_{R_{\mathrm{cr}}} = \left(\left(\frac{1}{4\beta\theta_o^3|_{R_{\mathrm{cr}}}}\right) - \frac{1}{2}\right)\frac{\theta_o^4|_{\mathrm{cr}} - \theta_\infty^4}{\theta_o^3|_{\mathrm{cr}}} \quad (15)$$

$$Q_{\max} = \frac{(1-\theta_o|_{\mathrm{cr}})}{1-2\beta\theta_o^3|_{\mathrm{cr}}} = \left(\frac{1}{4\beta\theta_o^3|_{\mathrm{cr}}}\right)\frac{\theta_o^4|_{\mathrm{cr}} - \theta_\infty^4}{\theta_o^3|_{\mathrm{cr}}} \quad (16)$$

The maximum heat transfer exists for $R_{\mathrm{cr}} > 1$ which means the dimensionless parameter β should be less than $\frac{1}{2\theta_o^3}$. Therefore, for the limiting case where $R = 1$ or $\beta = 0.5$ (i.e., zero coating thickness), the surface temperature and the heat transfer become

$$\theta_o|_{R=1} = 1 \quad \text{and} \quad Q|_{R=1} = 0.5(1-\theta_\infty^4) \quad (17)$$

Figure 3 shows the variation of the heat transfer as a function of the coating thickness for different values of β and $\theta_\infty = 0.8$. The locus of the maximum heat transfer values is added to the figure as a dashed line.

C. Case 3: Space Applications: ($Bi = 0$) and ($\theta_\infty = 0$)

In an environment with surroundings temperature of zero ($\theta_\infty = 0$) and no convection ($Bi = 0$), the critical coating thickness, according to Eq. (9), is the same as Eq. (14).

The maximum heat transfer and the surface temperature are determined by substituting $\theta_\infty = 0$ into Eqs. (15) and (16):

$$Q_{\max} = \frac{(1-\theta_o|_{\mathrm{cr}})}{1-2\beta\theta_o^3|_{\mathrm{cr}}} = \left(\frac{1}{4\beta}\right)\frac{1}{\theta_o^2|_{\mathrm{cr}}} \quad (18)$$

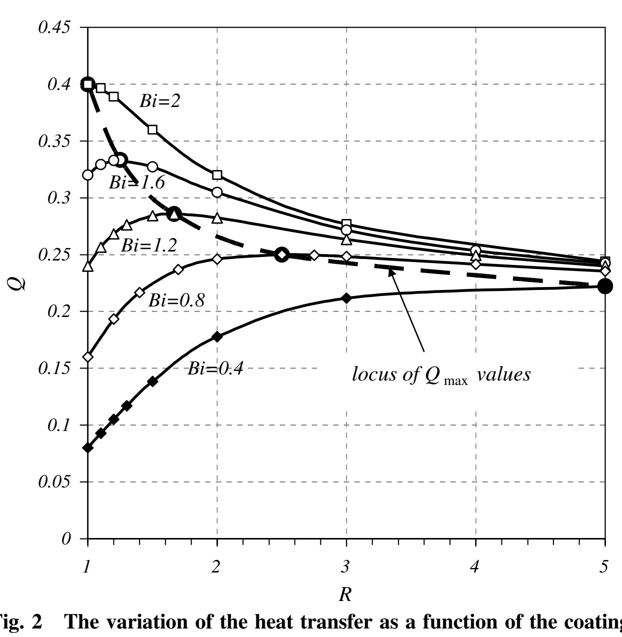

Fig. 2 The variation of the heat transfer as a function of the coating thickness for several cases of Biot Number (Bi) with no radiation ($\beta = 0$) and $\theta_\infty = 0.8$.

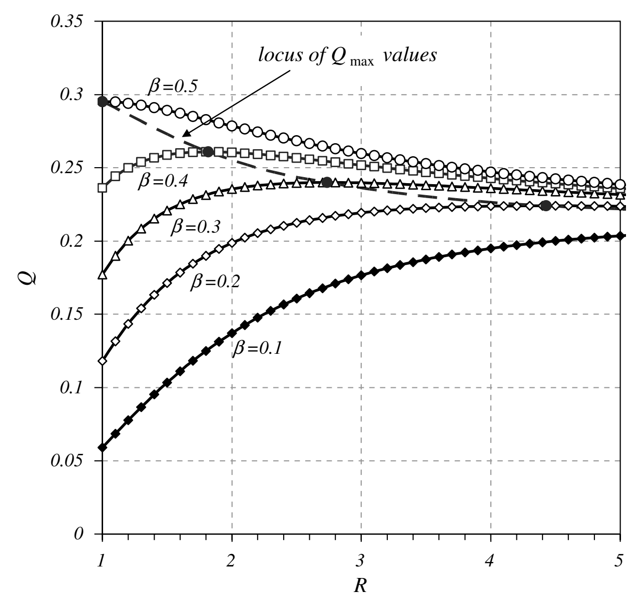

Fig. 3 The variation of the heat transfer as a function of coating thickness for several cases of dimensionless radiation parameter (β) with no convection ($Bi = 0$) and $\theta_\infty = 0.8$.

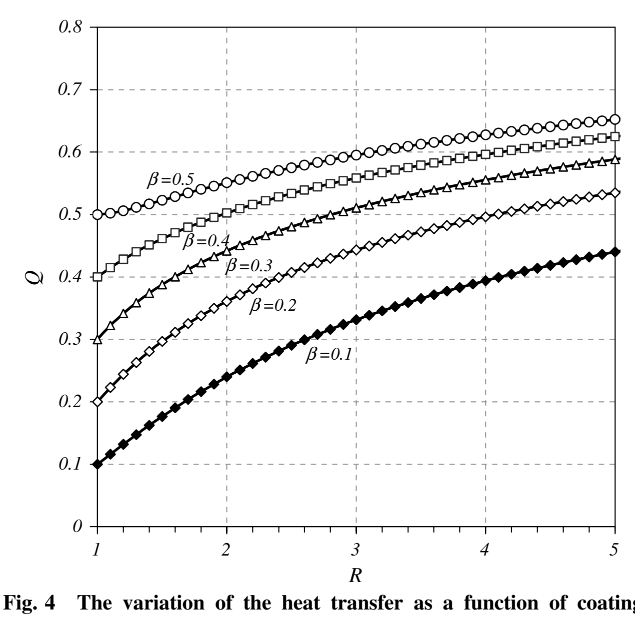

Fig. 4 The variation of the heat transfer as a function of coating thickness for several cases of dimensionless radiation parameter (β) with no convection ($Bi = 0$) and $\theta_\infty = 0$.

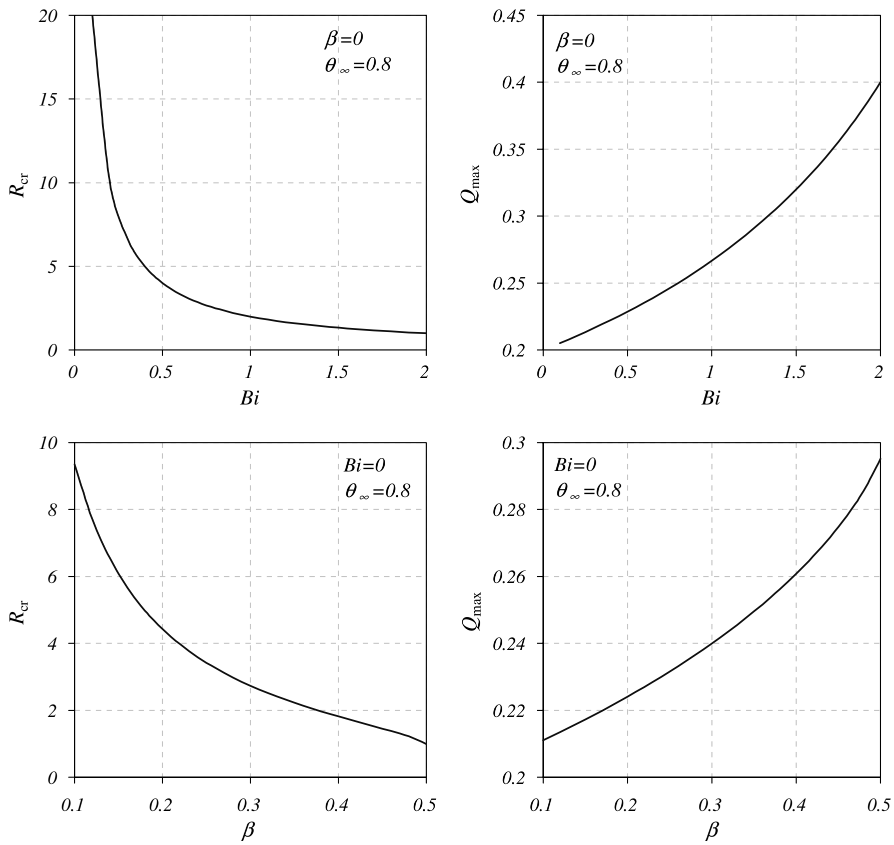

Fig. 5 The variation of the critical coating thickness and the maximum heat transfer as a function of Biot Number (*Bi*) and the radiation parameter (β).

$$\theta_o^3|_{R_{cr}} - 2\theta_o^2|_{R_{cr}} + \frac{1}{2\beta} = 0 \tag{19}$$

The maximum heat transfer exists for $R_{cr} > 1$ which means the dimensionless parameter β should be less than $\frac{1}{2\theta_o^3}$. Therefore, for the limiting case where $R = 1$ or $\beta = 0.5$ (i.e., zero coating thickness), the surface temperature and heat transfer in Eq. (17) become

$$\theta_o|_{R=1} = 1 \quad \text{and} \quad Q|_{R=1} = 0.5 \tag{20}$$

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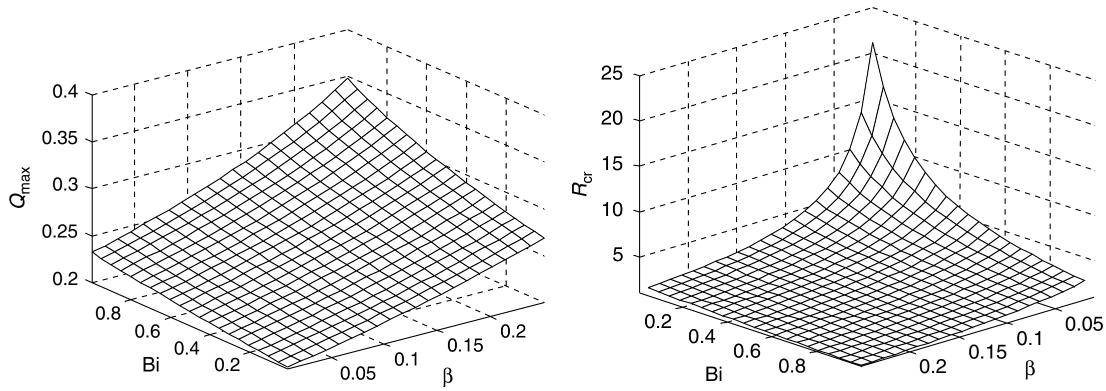
Figure 4 shows the variation of heat transfer as a function of coating thickness for different values of β and $\theta_\infty = 0.8$. The locus of the maximum heat transfer values was not included in the chart as with cases 1 and 2. because the maximum heat transfer is achieved at a large dimensionless radius $R = 987$. For aerospace applications, it is better to leave the system without insulation to achieve minimum heat transfer considering the need for lowest volume (or weight).

As found in all three of the above cases, the critical coating thickness decreases as Bi increases or as β increases, as shown in Fig. 5. Also, the maximum heat transfer increases as Bi increases or β increases.

D. Case 4: General Case

A numerical solution for $\theta_\infty = 0.8$ is shown in Figs. 6a and 6b, from solving Eqs. (6) and (7). Figures 6a and 6b show the variation of the critical coating thickness and the maximum heat transfer with respect to Bi and β. The critical coating thickness is found to increase as both convection and radiation decrease. On the other hand, the maximum heat transfer decreases as both convection and radiation decrease.



a) b)

Fig. 6 (a) The variation of the critical coating thickness as a function of *Bi*and β for $\theta_\infty = 0.8$, (b) The variation of the maximum heat transfer as a function of *Bi* and β for $\theta_\infty = 0.8$.

V. Conclusions

Because of the nonlinear nature of the problem, the solution for the critical radius has been obtained by numerical means, however, an explicit analytical solution has been obtained for three special cases.

The critical radius of the coating is found for a spherical particle which is subjected to a radiative and convective heat transfer environment. It is found that the critical radius of the coating increases when either convection or radiation decreases. Therefore, the existence of the critical radius depends on both the convection and radiation parameters. The critical coating radius increases for decreasing convection and radiation heat transfer. Thus, it becomes feasible to use a critical coating radius for heat transfer enhancement only for high convection or radiation heat transfer environments.

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